Algebraic relations between values of Mahler functions in positive characteristic

Gwladys Fernandes

Let $\mathbb{K}$ be a function field of positive characteristic, and let $d \geq 2$ be an integer. We say that a formal power series $f(z) \in \mathbb{K}[[z]]$ is d-Mahler over $\mathbb{K}(z)$ if there exist polynomials $P_0(z), \ldots, P_n(z) \in \mathbb{K}[z]$, $P_n(z) \neq 0$, such that:

$$P_0(z)f(z) + P_1(z)f(z^d) + \cdots + P_n(z)f(z^{d^n}) = 0.$$

Let $f_1(z), \ldots, f_n(z)$ be d-Mahler functions and let α be an algebraic number over $\mathbb{K}$. We are going to present the following result, obtained during our thesis: Under some assumptions we will explain, every homogeneous algebraic relation over $\overline{\mathbb{K}}$ between $f_1(\alpha), \ldots, f_n(\alpha)$ arises from the specialization at $z = \alpha$ of a homogeneous algebraic relation over $\overline{\mathbb{K}}(z)$ between $f_1(z), \ldots, f_n(z)$. When $\mathbb{K}$ is a number field, this result is established by B. Adamczewski and C. Faverjon, as a consequence of a statement of P. Philippon.

We will mention the connexions this framework shares with different areas of mathematics, as finite automata and Drinfeld modules, along with the perspectives of the study of the algebraic relations between the Mahler functions themselves.